F0-Liner heating due to the resistive wall impedance

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The circulating proton beam has to lose some energy in the liner wall due to the resistive component of longitudinal impedance. This effect can lead to heating of the liner and should be taken into account parallel with heating by the lost protons during injection. The loss factor and corresponding power loss per unit of the liner length can be defined in the forms:

$$\frac{\partial k_{\Rightarrow}}{\partial z} = \frac{\Delta E}{\left(N_{ppb} \cdot e\right)^2} \qquad \frac{dP}{\partial z} = \frac{\partial k_{\Rightarrow}}{\partial z} \cdot \frac{I_0^2}{f_0 n_b},$$

where ΔE and $\left(N_{ppb} \cdot e\right)$ are the total energy change and the total charge of the bunch ;
n_b is the number of proton bunches;
$I_0 = n_b \cdot N_{ppb} \cdot e \cdot f_0$ is the average circulating proton current.
For the Gaussian longitudinal particle distribution in the form:

$$I(t) = \cdot \frac{N_{ppb} \cdot e}{\sqrt{2p}\, s_t} e^{-t^2 / 2s_t^2}$$

the Fourier transform of the current distribution is given by:

$$I(w) = \left(N_{ppb} \cdot e\right) e^{-w^2 s_t^2 / 2}$$

The integral of the energy loss factor due to the resistivity of the liner wall can be written as follows:

$$\frac{\partial k_{\Rightarrow}}{\partial z} = \frac{1}{\left(N_{ppb} \cdot e\right)^2} \int_{-\infty}^{+\infty} \mathrm{Re}\left[\frac{\partial Z_{\Rightarrow}(w)}{\partial z}\right] \cdot I^2(w) \cdot \frac{dw}{2p} = \int_{-\infty}^{+\infty} \mathrm{Re}\left[\frac{\partial Z_{\Rightarrow}(w)}{\partial z}\right] \cdot \left(e^{-w^2 s_t^2 / 2}\right)^2 \cdot \frac{dw}{2p}$$

A longitudinal impedance per unit length for the round cross-section beam pipe is given by:

$$\frac{\partial Z_{\Rightarrow}(w)}{\partial z} = \left[1 + j \cdot \mathrm{sgn}(w)\right] \frac{r}{pd \cdot b} \cdot F_g,$$

where $d = \sqrt{\frac{2 r c}{|w| Z_0}}$ is the skin depth, b is the pipe radius, r is the wall specific resistivity, $z_0 \approx 377\ \Omega$ is the vacuum impedance, and F_g is a geometric form-factor ($F_g = 1$ for the round shape).
Above-mentioned equation can be applied for calculation of the Lambertson liner longitudinal impedance, provided that its real shape is approximated by a semi-cylinder with the effective radius b_{eff} and with $F_g = 0.5$.

Because the bunch-current spectrum density is nonzero only in the frequency range up to wavelength of the order of the bunch length $w_b = c/s_l$, the higher frequencies do not contribute significantly to the energy loss. Therefore, the integral for the loss factor can be evaluated analytically integrating over the frequency spectrum in approximation of $w_b << w_a$:

$$\frac{\partial k_{\Rightarrow}}{\partial z} = \frac{1}{4\sqrt{2}p^2 s_l^{3/2}} \frac{cF_g \cdot \Gamma(3/4)}{b_{eff}} \sqrt{Z_0 r}$$

where $\Gamma(3/4) \approx 1.23$ is a Gamma-function, $s_l = c s_t$ is the rms longitudinal beam size, w_a is the upper frequency limit within of which the classical skin effect is valid. Typically $f_a = w_a / 2p > 1\,GHz$ when $f_b = w_b / 2p \cong 50\,MHz$.

The total power absorbed by wall of the Lambertson liner with length L_{tot} is:

$$P_{liner} = \frac{I_0^2 \cdot L_{tot}}{4\sqrt{2}p^2 s_l^{3/2} f_0 n_b} \frac{cF_g \cdot \Gamma(3/4)}{b_{eff}} \sqrt{Z_0 r} \qquad [Watt]$$

At the injection energy E=150 GeV and central orbit, estimation of the power loss in the Cu-Be liner wall for the multi-bunch operation mode 36 x 0 with $N_{ppb} = 3 \cdot 10^{11}$ and $I_0 = 82\,mA$ results in:

$$P_{tot} \approx \frac{(0.082)^2 \cdot (Amps)^2\, 12\, m}{4\sqrt{2}(3.1416)^2\, 0.9\, m \sqrt{0.9\, m}\, 47.7 \cdot 10^3 \sec^{-1} \cdot 36} \frac{2.97 \cdot 10^8 m \cdot \sec^{-1} \cdot 0.5 \cdot 1.23}{9 \cdot 10^{-3}\, m} \sqrt{377\,\Omega \cdot 6 \cdot 10^{-8}\,\Omega \cdot m} \approx 0.095 \quad [Watt]$$

$s_l = 0.9\,m$, $r_{Cu-Be} = 6 \cdot 10^{-8}\,\Omega \cdot m$, $b_{eff} = 9\,mm$

The power loss per unit of the liner length:

$$\frac{dP}{dl} \approx 0.008 \quad [Watt/m]$$

Since the power loss in the liner wall is inversely proportional to the longitudinal beam size as $P \succ 1/s_l^{3/2}$ and, in turns, $s_l \succ 1/E^{1/4}$ that Tevatron energy scaling law for the liner heating can be defined as follows:

$$P \succ E^{3/8}$$

At the collision energy E=980 GeV this gives a factor of two as compared with the injection energy.

$$P_{tot}(980) \approx 0.19 \quad [Watt]$$

The liner heating due to the resistive wall impedance is expected to be small and it couldn't affect on an equilibrium temperature.